

Recent Results in String Duality

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This is a talk given at YKIS '95, primarily to non-string theorists. I review the evidence for string duality, the principle that any string theory at strong coupling looks like another string theory at weak coupling. A postscript summarizes developments since the conference.

When I gave this talk in late August I resolved to write it up immediately, because the field was moving very fast and the ‘recent’ results would quickly become dated. Well, as I sit down to write it is actually two and a half months later, and indeed many things have happened since my talk. So I will write up the talk as given, and add a postscript to describe a few of the developments since.

§1. Introduction

Until recently, one might have expected that string theories at strong coupling would involve new and exotic physics. For example, since string theory includes gravity, one might have expected a phase with large fluctuations of the spacetime geometry. But now it appears that string theory at strong coupling is not so exotic. Rather, *string duality* is the principle that any string theory at strong coupling simply looks like another string theory at weak coupling.

This sort of duality is familiar in low dimensional field theories. That it might happen in string theory has been put forward by a number of people over the years, and was extensively pursued by Duff, Sen, Schwarz, and others. Until recently though, it seemed unlikely to many of us that such a thing could happen even in a nontrivial four dimensional field theory, and much more unlikely that it could happen in a theory as complicated as string theory. But the work of Seiberg and others has made it clear that this can indeed happen in four-dimensional gauge theories, and now it appears overwhelmingly likely that it happens in string theory as well.

String duality is a revolution, shaking our understanding of the foundations of the theory. Many of the world-sheet properties that previously received great emphasis now appear to be technical features of string perturbation theory, not preserved by duality. On the other hand, spacetime ideas such as supergravity are playing a more prominent role in determining the structure of the theory. At this point it is not clear where we are headed or whether the final result will still be called ‘string theory.’ The main theme of my talk is “Should you believe it?” Along

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the way I will try to discuss a few of the open questions and some of what has been learned.

Here is a list of a few of the conjectured dualities, in various numbers of dimensions:

$$\begin{aligned}
 d = 10 : \quad & \text{IIA string} \leftrightarrow \text{'M-theory' on } S_1 \\
 & SO(32) \text{ heterotic string} \leftrightarrow SO(32) \text{ type I string} \\
 d = 7 : \quad & \text{heterotic string on } T^3 \leftrightarrow \text{M-theory on } K3 \\
 d = 6 : \quad & \text{heterotic string on } T^4 \leftrightarrow \text{IIA string on } K3 \\
 d = 4 : \quad & \text{heterotic string on } T^6 \leftrightarrow \text{heterotic string on } T^6
 \end{aligned} \tag{1}$$

This is a remarkable list: on the one hand it looks quite complicated—in each of the listed dimensions the heterotic string is dual to a different theory. But the structure is quite constrained, and fits together in an intricate way as one compactifies and decompactifies dimensions. By the way, all the theories on this list have a large amount of supersymmetry, the equivalent of $N = 4$ in four dimensions. For theories with less supersymmetry the dynamics and dualities are even richer.

A skeptic might take the point of view that all evidence for string duality is circumstantial, and that the successes of string duality are consequences only of the strong constraints that supersymmetry imposes on the low energy field theory. A severe skeptic might make this same argument in regard to weak/strong duality in $N = 4$ supersymmetric *field* theory, while a less severe one might believe that duality is possible in field theory but does not extend to the full string spectrum. Both of these points of view were reasonable at one time (or at least I hope they were, because I held them), but the evidence has rapidly mounted, to the point that the issue has shifted from “is it true?” to “what does it mean?”

In this talk I will try to assemble the evidence. This will be far from a systematic review of the subject, but rather a presentation of those issues to which I have given the most thought.

§2. Evidence for String Duality

2.1. *Heterotic S-Duality in $d = 4$.*

A great deal of attention has been given to the heterotic string compactified on a 6-torus. This is supposed to have a infinite discrete symmetry $SL(2, Z)$, generated by a weak/strong duality transformation combined with discrete shifts of the θ -parameter¹⁾. The evidence, reviewed in ref. 2), is very similar to that for the older conjecture of duality in $N = 4$ supersymmetric field theory: invariance of the BPS mass formula for stable electric and magnetic charges, and of the lattice of charges allowed by the Dirac quantization condition. This is not convincing to either skeptic: it could just be a consequence of supersymmetry. It would be less trivial, and say more about the dynamics, if one could show that the *actual* spectrum of BPS states (the degeneracies of the allowed states) is dual. There is some evidence here, the existence of a monopole bound state required by string duality. But this is only

evidence for duality in *field* theory. A search for the duals of string states has been inconclusive, depending on an understanding of soliton collective coordinates at the string scale.³⁾

2.2. Heterotic String on $T^4 \leftrightarrow$ IIA String on $K3$

In six dimensions, the heterotic string on a 4-torus and the Type IIA string on $K3$ (the only 4-dimensional Calabi-Yau manifold) have the same low energy field theory and in fact the same space of vacua, the coset space

$$O(A_{4,20}) \backslash O(20, 4; R) / O(20, R) \times O(4, R). \quad (2)$$

Low energy supersymmetry by itself requires that the space locally be of the form

$$O(n, 4; R) / O(n, R) \times O(4, R). \quad (3)$$

So the fact that $n = 20$ in both cases is a nontrivial coincidence.⁴⁾ That the global structures (the discrete identifications) match is a further nontrivial fact, involving stringy states and not just the low energy spectrum: the discrete identification on the heterotic side includes T -duality, interchanging winding and momentum states, while the discrete identification on the IIA side includes mirror symmetry and discrete shifts of world-sheet θ -parameters.⁵⁾

A possible contradiction arises because the IIA string has an Abelian gauge group $U(1)^{24}$, while the heterotic string has this group in generic vacua but has unbroken non-Abelian symmetries at special points. The mechanism to resolve this is the same as that found by Strominger for the conifold⁶⁾ (discussed further below): when a nontrivial surface in the compact dimensions shrinks to zero size, a wrapped soliton can become massless, providing the needed gauge bosons. Indeed, Witten⁷⁾ argued that the $K3$ theories with enhanced gauge symmetry contained collapsed spheres. There was still a puzzle because these $K3$'s are orbifolds and so correspond to solvable and nonsingular conformal field theories, where string perturbation theory should be a good qualitative guide. The existence of a massless soliton, a large nonperturbative effect, would then be surprising and disturbing. It would mean that perturbation theory can break down without warning, a severe problem since we have no other definition of string theory. Again this is evaded: Aspinwall showed that the orbifold and enhanced-symmetry theories differ by a background antisymmetric tensor field, so it is quite possible that the latter theory corresponds to a singular CFT as is the case for the conifold.

2.3. The Big Picture

In his famous talk at USC,⁷⁾ Witten proposed that every string theory in every dimension has a strong coupling dual.*⁸⁾ Collecting some earlier duality conjectures, discarding others, and adding some of his own, he produced a nearly complete set of duals (much of this list was anticipated in ref. 8)). A skeptic could argue that most of this paper is based on low energy field theory and on unproven conjectures

*) To be precise, the conjecture holds in this simple form only for theories with at least $N = 4$ supersymmetry, where the BPS formula allows a global definition of the dilaton.

about the existence of BPS states. But there are many nontrivial checks in it. In particular, although the pattern of duals has an intricate dependence on dimension (see the earlier list) compactification never leads to two *different* weakly coupled dual candidates for a given theory, which would be a contradiction. There is always exactly one candidate. Moreover, for all the dual pairs, the field redefinition that relates the low energy theories always includes a sign change of the dilaton (else one would have a weak/weak duality of different string theories, again a contradiction).

2.4. *The Conifold Transition*

For several years there has been a puzzle that certain Calabi-Yau compactifications are singular,⁹⁾ with various couplings diverging. Although these couplings are calculated at string tree level, for the Type II string there are nonrenormalization theorems so that in some cases (the vector multiplet Kahler potential) the result is known to be free of perturbative *and nonperturbative* corrections.⁶⁾ That is, the exact effective action is singular. This was shown to have a simple and natural interpretation in terms of a massless soliton (see above). But in some cases¹⁰⁾ there is a new branch of vacua in which the soliton has an expectation value. This highly nonperturbative phase has a natural dual description in terms of a weakly coupled string theory on a Calabi-Yau space of different topology.

Incidentally, an important open question is to understand the precise nature of the breakdown of perturbation theory at the conifold and enhanced gauge symmetry points. There are interesting conjectures here^{11), 12)} but no clear picture.

2.5. *Heterotic String as a Soliton*

In addition to light states from small loops, string theories include strings of macroscopic size. An infinite straight string (or a string wound around a large periodic dimension) is a BPS state, invariant under half the supersymmetry, and so is stable under changes in the parameters. Taking the example of the heterotic string theory in six dimensions, start with a macroscopic heterotic string in the weakly coupled theory. Increase the coupling until the strong coupling limit, described by the weakly coupled IIA string, is reached. A state which looks like a long heterotic string must still exist but is no longer present as a fundamental string state: it must be a soliton.

Indeed, the IIA theory contains a soliton with exactly the properties of the heterotic string.¹³⁾ In particular, the degrees of freedom of the soliton are precisely those of the fundamental string:¹⁴⁾ transverse oscillations moving on the torus of the dual heterotic theory, right-moving spinors and a left-moving $E(8) \times E(8)$ current algebra. This is a necessary check on string duality, and is a strong piece of positive evidence. Start from the weakly coupled IIA theory, in which this heterotic soliton is heavy. As the coupling is increased the BPS formula implies that the soliton becomes lighter, and in the strong coupling limit its tension is the smaller than any other scale: it is hard to see how the effective theory could then be anything but the heterotic string.*)

*) It should be noted that the converse check, finding the IIA string as a soliton in the heterotic theory, is less clear-cut. A soliton carrying the appropriate charge exists—this by itself is

2.6. Loop and Nonperturbative Corrections

Direct checks of weak/strong duality are difficult because one can only calculate in the weakly coupled theory. But in theories with at least $N = 2$ supersymmetry, there are nonrenormalization theorems which guarantee that some amplitudes calculated at weak coupling are in fact exact in the quantum theory. The strategy is a common one in string theory: the coupling, and therefore the quantum corrections, depend on the dilaton and supersymmetry restricts the way the dilaton can appear in the effective Lagrangian. With $N \geq 4$ supersymmetry there are a few couplings that can be compared in this way,¹⁵⁾ but the case $N = 2$ is particularly rich: an infinite number of couplings (an entire function) can be compared; for a review see ref. 16). This appears to be a very nontrivial check, relating string tree and loop calculations, as well as nonperturbative results, on the two sides.

An important open question is to understand the strong-coupling behavior of $N = 2, 1$, and 0 theories as completely as that for $N = 4$. Their richer dynamics makes these theories more interesting (and of course we live in an $N = 0$ or approximately $N = 1$ vacuum), but also makes it harder to solve them. Another key question is how string duality relates to all the recent results on supersymmetric gauge theories. Obviously there must be a close connection, but as yet the two subjects are surprisingly disjoint. For example, in the gauge theories the focus is on the physics at long distance, whereas string duality appears to hold at all scales.

2.7. CHL Models

The final check I will describe in more detail, not because it is especially important but because it is the one in which I have personally been involved, and serves to illustrate some important ideas. Let me first recall the beautiful work of Narain,¹⁷⁾ who described the space of vacua of the toroidally compactified heterotic string. This displays several important phenomena in string theory: the existence of a moduli space of degenerate but physically inequivalent vacua, a discrete group of equivalences (dualities), points of enhanced gauge symmetry, and limits (decompactifications) in which the theory goes over to seemingly different ten-dimensional string theories, the $E(8) \times E(8)$ and $SO(32)$ heterotic strings. Note that all of this is perturbative, holding in the weakly coupled theory—it is not the nonperturbative string duality that is currently causing so much excitement. But in fact a central theme of string duality is that more of these same phenomena arise nonperturbatively in the larger moduli space that includes the string coupling (dilaton). In fact, the dilaton, which plays such an important role in string perturbation theory, is not distinguished in the full quantum theory: it is on an equal footing with the moduli from the compactification.

Strong coupling duals for these $N = 4$ theories were proposed in refs. 2), 8), 7). It was widely assumed that these were the only $N = 4$ vacua of the heterotic string,^{*)} but Chaudhuri, Hockney, and Lykken (CHL)¹⁹⁾ pointed out the existence of many

rather trivial—but its degrees of freedom are harder to determine, as they involve subtleties in the quantization of solitons below the string scale.

*) Except for some unpublished work of Dixon, quoted in the review 18).

new $N = 4$ vacua. It turns out that all of these can be identified with toroidal compactifications of the heterotic string,^{20), 21)} but with fields periodic only up to a discrete global symmetry such as the interchange of the two $E(8)$'s of the heterotic string; Narain's work had included periodicity up to a *local* symmetry.

Considering first $d = 4$, the Narain compactifications are supposed to be self-dual under weak/strong duality (S -duality).^{1), 2)} For the CHL theories there is a complication. Narain compactification gives simply-laced gauge groups ($SU(n)$, $SO(2n)$, $E(n)$), but the CHL theories include non-simply laced groups. The moduli space was explored in ref. 21) and was found to include points with symmetries

$$Sp(20) \times SO(17-2d), \quad Sp(18) \times SO(19-2d), \quad \dots, \quad Sp(2d) \times SO(37-4d). \quad (4)$$

Now, S -duality includes electric/magnetic duality in the low energy theory. This takes simply-laced groups into themselves, but interchanges long and short roots and so takes $Sp(2n) \leftrightarrow SO(2n+1)$.²²⁾ For Narain compactifications, S -duality acts trivially on the moduli, but in the CHL theories it must act on the moduli so as to move the theory from a point of one symmetry to a point of dual symmetry. Now there is a nontrivial check, because gauge groups had better appear in the moduli space in dual pairs. This need only hold in $d = 4$, because only in this case is there electric/magnetic duality: only in four dimensions do the field strength $F_{\mu\nu}$ and its dual have the same rank (this also gives some idea as to why the pattern of string dualities depends so strongly on dimension). Examining the list (4), we see that for general d the set is not dual, but precisely in $d = 4$ it is. Nothing in the compactification distinguishes $d = 4$, and duality of the groups is not automatic in some trivial way, but precisely in $d = 4$ where it must appear it does.

One might wonder whether this is an independent check. In fact it can almost be derived from facts we already know, but the derivation is a nice illustration of how the whole pattern of string dualities fits together. To start, let us ask how it is that the toroidally compactified heterotic string is dual to itself in $d = 4$ but to the IIA string in $d = 6$: how do these fit together if we compactify the $d = 6$ theory on T^2 ? The answer is that the group of dualities is actually very large, with a given string theory generally having many self-dualities (both perturbative and non-perturbative) as well as equivalences to other string theories. But at a given point in parameter space at most one of this set will have a weak coupling and a natural spacetime interpretation,^{*)} which is one of the checks mentioned before. Which theory is weakly coupled is determined by d -dependent dimensional analysis,⁷⁾ leading to the intricate pattern (1).

If two string theories are equivalent, their self-duality groups must be conjugate: 'duality of dualities.'²³⁾ In particular, the nonperturbative S -duality of the $d = 4$ heterotic string is conjugate to the perturbative T -duality of the IIA string on $K3 \times T_4$:

$$S = \beta T \beta^{-1} \quad (5)$$

where β stands for the $d = 6$ string-string duality. The non-perturbative S -duality

^{*)} The latter requirement excludes compactification radii shorter than the string scale and so factors out the perturbative T -dualities.

of the $d = 4$ heterotic string thus follows from the β symmetry. The same is in principle true of the S -duality of the CHL theories.^{24)*}

In $d = 6$ the CHL theories present a puzzle: here the Narain theories are dual to the IIA theory on $K3$, but there is no other $N = 4$ IIA compactification in conformal field theory. This was elegantly resolved in ref. 25), which proposed a dual with a nontrivial Ramond-Ramond background field. Such a background cannot be described in conformal field theory and is not well-understood as yet. The arguments in ref. 25) use another strategy common in string theory. Start from the $d = 11$ M-theory compactified on a product $K3 \times S_1$. Consider compactification first on $K3$ and then on S_1 , and then in the reverse order. Using the duality conjectures (1) for compactification of the M-theory, one obtains a known dual pair, the heterotic theory on T^4 and the IIA theory on $K3$. Now adding a twist to the compactification (identifying under a combined shift on the S_1 and isomorphism on the $K3$), one obtains the dual pair of ref. 25), one of which is the $d = 6$ CHL theory.

§3. Discussion

The picture that emerges from all of this is of a single theory, with a space of vacua labeled by the dilaton and the moduli from the compactification. In the bulk of moduli space the theory is fully quantum mechanical, but it has many classical limits. The asymptotics in the various classical limits are given by string perturbation theory, and all of the seemingly different supersymmetric string theories appear as limits of this single theory.

Coming back to the question “Should you believe it?” and speaking as a former skeptic, I would say that my skepticism ended around section 2.5, the heterotic string as a soliton. Too many things work. Test after test that could have failed succeeds, to the point that the simplest explanation is that all of these theories are connected.

The question now is, “What is the theory?” Is it even a string theory? One point of view would be that all of the various string theories can be formulated nonperturbatively as string field theories, and then transformed into one another by an appropriate change of variables. This is parallel to what is possible in the Thirring/Sine-Gordon duality, the prototype of a quantum-mechanical equivalence. However, I think that the evidence points in another direction, namely that the various string theories are just generators of the asymptotics of the theory at weak coupling and below the Planck scale. Even before string duality there were reasons to believe this, such as the limited success of string field theory, and the rapid growth of string perturbation theory.²⁶⁾ In the matrix model for example, the rapid growth of string perturbation theory is a sign that the string are best thought of as a composite,²⁷⁾ in that case of free fermions. String duality gives a further insight: strings appear in the various limits simply because the BPS formula requires them to be the lightest objects in the theory, and not necessarily because they play a

*) Ref. 24) does not find the full S -duality group, in particular not the transformations which act nontrivially on the moduli. These must involve a product of T and a symmetry acting on the $K3$ moduli space.

fundamental role.

In some ways the situation is much like the 60's and early 70's, with a great deal of 'data' and no theory. We know a lot about the asymptotics of the theory, and a bit about how they fit together. An interesting clue is the strong coupling limit of the $d = 10$ IIA string, which appears to be an eleven-dimensional theory. There is no perturbative string theory in eleven dimensions. Evidently the long distance physics is $d = 11$ supergravity, but the short distance physics is unknown and for now is referred to as 'M-theory.' This theory has no coupling constant (dilaton) and so is intrinsically quantum mechanical.

Does string duality help address the phenomenological problems of string theory, the cosmological constant and the choice of vacuum? As yet the results are disappointing, in that there is no new physics at strong coupling but just more of what has been seen at weak coupling.*⁾ But it is not time to be impatient. We are learning remarkable and surprising things about string theory. That test after test of string duality is working tells us that there is a new structure to be found. Once it is found we may hope that we will learn new things about the dynamics of string theory and the structure of the vacuum. Indeed, it would be disappointing if we were able to find the right string 'model' without first answering the question, "What is string theory?"

§4. Postscript

The field has continued to move at a rapid rate. We still have not answered the question posed at the end of the previous section, but the web of connections between the different string theories has gotten much tighter. Some of the developments:

4.1. *D-Branes*

In closed string theories, a string state can be roughly factored into the separate states of the right-moving and left-moving degrees of freedom. The type II string thus has two kinds of boson, the Neveu-Schwarz/Neveu-Schwarz (NS NS) states which are products of bosons, and the Ramond/Ramond (RR) states which are products of fermions. Both sectors include gauge fields, and in fact the RR sector includes generalized gauge fields, antisymmetric tensors of various ranks. Fundamental strings carry only the NS NS charges, but string duality requires states in the spectrum which carry RR charge as well. Previously^{8), 7), 6)} the necessary states were described as black holes. This was not entirely satisfying, however, because of the singularity and the difficulty of quantization.

It was argued some time ago²⁸⁾ that in addition to fundamental string states, string theory necessarily contained 'D-branes,' extended objects swept out by the endpoints of open strings. It turns out²⁹⁾ that these objects also carry the RR charge. The quantum is precisely that required by the Dirac quantization condition, and by string duality. Thus it seems that these are the RR-charged objects need for string duality. This is a more precise description than the black hole, at least for

*⁾ See however the final section of ref. 11).

states of small charge. Such a simple description is apparently possible because the RR-charged objects are lighter than ordinary solitons (one less power of the coupling) and so are in some sense a smaller disturbance (analogous to the single-eigenvalue tunneling in the matrix model).

The identification of the RR solitons as D-branes has made suddenly possible many explicit studies of the spectrum, in agreement with duality.³⁰⁾ At the same time, however, it points up even more than before that we are only working with an effective theory. A sum over all virtual D-branes would be extremely unwieldy and is unlikely to be correct. Rather, the D-brane description is likely valid only at scales long compared to that set by the mass (or tension) of the D-brane, so such a sum is inappropriate.*) There is an interesting question as to the physical size of D-brane, and of the fundamental degrees of freedom that we are looking for. Shenker has suggested that there is evidence for a scale shorter than the string scale.³¹⁾ This is at first sight plausible, as the Planck scale now seems to play a more central role than the string scale. But I do not know of any sense in which the effective size of the D-brane is smaller than the string scale. See refs. 32) for further investigation of this.

4.2. *Type I–Heterotic Duality*

Of the various dualities conjectured in ref. 7), the $d = 10$ $SO(32)$ type I–heterotic duality was somewhat separate from the others, and had the least supporting evidence. Subsequently a heterotic soliton was found in the type I theory, but it is of a particularly singular sort so one cannot be sure that it is in the spectrum.³³⁾ Indeed, the strongest argument for this duality was simply “What else?”—given the evidence in other cases that string duality is a general principle, this is the natural pairing to make in $d = 10$.

There is now new evidence.³⁴⁾ First, following the idea of ‘duality of dualities,’ we can ask how the T -dualities of the compactified heterotic string map to the type I theory. The answer is that they imply a rather complicated self-duality of the type I theory, one which does not hold in perturbation theory. This is an apparent contradiction, because the type I theory seems to be weakly coupled in the relevant region. But a more careful examination shows that perturbation theory breaks down in a novel and intricate way, at the precise point in parameter space where string duality requires light nonperturbative states to appear. Second, the heterotic soliton in the type I theory carries RR charge. As noted above, it now appears that such solitons should be described as D-branes. The relevant D-brane has precisely the world-sheet structure of heterotic string.

4.3. *The $E(8) \times E(8)$ Theory*

A notable gap in ref. 7) was the absence of a candidate dual for the $d = 10$ $E(8) \times E(8)$ heterotic string. This gap has now been filled. Compactifying the $d = 11$ M-theory on a circle gives the IIA string. Ref. 35) elegantly argues that compactifying the same theory on a line segment produces the $d = 10$ $E(8) \times E(8)$ heterotic string, with the gauge symmetry living on the boundaries of spacetime.

*) Except in the special circumstance that a D-brane is light due to a degenerating cycle.

This is further evidence for the relevance of the $d = 11$ theory as well.

4.4. Conclusion

Again and again the predictions of string duality are borne out, often in surprising ways. It appears that at present the theory is smarter than we are, knowing how to connect many disjoint pieces of physics and mathematics. It remains to unravel the hidden structure that makes this possible. It is notable that many interesting but separate lines of development from recent years—mirror symmetry, supergravity, supersymmetric solitons, D-branes, and others—have now come together to play key roles in string duality. It is likely that there are other past developments whose significance has not yet been realized, and which will in their turn help to complete the picture.

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